AP Statistics Chapter 6 Practice Test
Random Variables

Name Settle
Per __ Date ____________

Part 1: Multiple Choice. *Circle the letter corresponding to the best answer.*

For problems 1 and 2: Patients receiving artificial knees often experience pain after surgery. The pain is measured on a subjective scale with possible values of 1 (low) to 5 (high). Let X be the pain score for a randomly selected patient. The following table gives part of the probability distribution for X.

X	1	2	3	4	5
P(X)	0.1	0.2	0.3	0.3	0.1

d **1.** What is the probability that a randomly chosen patient has a pain level of three or higher?
(a) 0.40 (b) 0.60 (c) 0.3 (d) 0.7 (e) 1

a **2.** What is the expected value of pain?
(a) 3.1 (b) 2.5 (c) 1.5 (d) 0.2 (e) 3.5

$1(0.1)+2(0.2)+3(0.3)+4(0.3)+5(0.1)$

For problems 3 and 4: A random variable *X* has a probability distribution as follows:

X	3	4	5	6
P(X)	1b	2b	3b	4b

$1b+2b+3b+4b=1$
$10b=1$
$b=1/10$ or 0.1

Where b is a positive constant.

d **3.** The value of the constant b is
(a) 1.0 (b) 0.25 (c) 0.15 (d) 0.10 (e) 0.20

e **4.** The probability $P(X < 5.0)$ is equal to
(a) 1.0 (b) 3.0 (c) 0.60 (d) 0.15 (e) 0.30

$P(3)+P(4) = 0.1+0.2$

a **5.** In a game of 4-spot Keno, the player picks 4 numbers from 1 to 80. The casino randomly selects 20 winning numbers. The probability that a person wins \$0 is 0.741, \$1 is 0.213, \$3 is 0.043, and \$120 is 0.003. Find the expected amount of money won.
(a) 0.702 (b) 0 (c) 1 (d)0.25 (e) \$25.20

$0(0.741)+1(.213)+3(.043)+120(.003)$

d **6.** Roll one 6-sided die 8 times. The probability of getting exactly 3 ones in those 8 rolls is given by

n, K, $p=1/6$

(a) $\binom{6}{3}\bullet\left(\frac{1}{6}\right)^3\bullet\left(\frac{5}{6}\right)^5$ (b) $\binom{6}{3}\bullet\left(\frac{1}{6}\right)^5\bullet\left(\frac{5}{6}\right)^3$

(c) $\binom{8}{3}\bullet\left(\frac{1}{6}\right)^3\bullet\left(\frac{5}{6}\right)^3$ (d) $\binom{8}{3}\bullet\left(\frac{1}{6}\right)^3\bullet\left(\frac{5}{6}\right)^5$

(e) $\binom{8}{3}\bullet\left(\frac{1}{6}\right)^5\bullet\left(\frac{5}{6}\right)^3$

$\binom{n}{K}p^K(1-p)^{n-K}$

$\binom{8}{3}(1/6)^3(5/6)^{8-3}$

_____ **7.** X and Y are independent random variables, and *a* and *b* are constants. Which one of the following statements is true?

(a) $\sigma_{X+Y} = \sigma_X + \sigma_Y$ can't add standard deviations
(b) Var (X - Y) = Var (X) + Var (Y) ✓
(c) Var (a + bX) = b Var (X) → true for standard deviation not variance
(d) $\sigma_{X-Y} = \sigma_X - \sigma_Y$
(e) Var (X + Y) = $\sqrt{Var(X^2)+Var(Y^2)}$ → $\sigma_{x+y} = \sqrt{\sigma_x^2+\sigma_y^2}$

d 8. The variance of sum of two random variables X and Y is

~~(a)~~ $\sigma_X + \sigma_Y$

(b) $(\sigma_X)^2 + (\sigma_Y)^2$.

~~(c)~~ $\sigma_X + \sigma_Y$, but only if X and Y are independent.

(d) $(\sigma_X)^2 + (\sigma_Y)^2$, but only if X and Y are independent.

(e) None of these.

$\sigma^2_{x+y} = \sigma_x^2 + \sigma_y^2$

Var

Key

a 9. Let the random variable X represent the score of a random student on the Iowa Test of Basic Skills. Assume that X is Normal with mean 6.8 and standard deviation 1.6. What is $P(X > 9)$?

(a) 0.0846 (b) 0.9154 (c) 0.9031 (d) 0.7881 (e) 0.0969

normal cdf (9, 100, 6.8, 1.6)

lower upper bound mean std dev

c 10. A dealer in the Sands Casino in Las Vegas selects 20 cards from a standard deck of 52 cards. Let Y be the number of diamonds in the 20 cards selected. Which of the following best describes this setting?

$p = \frac{13}{52} = \frac{1}{4}$

(a) Y has a binomial distribution with $n = 20$ observations and probability of success $p = 0.25$.

(b) Y has a binomial distribution with $n = 20$ observations and probability of success $p = 0.25$, provided the deck is shuffled well.

(c) Y has a binomial distribution with $n = 20$ observations and probability of success $p = 0.25$, provided that after selecting a card it is replaced in the deck and the deck is shuffled well before the next card is selected.

independence

(d) Y has a geometric distribution with $n = 20$ observations and probability of success $p = 0.25$.

(e) Y has a geometric distribution with n = 52 observations and probability of success $p = 0.25$.

_____ 11. A set of 10 cards consists of 3 red cards and 7 black cards. The cards are shuffled thoroughly and you turn cards over, one at a time, beginning with the top card. Let Y be the number of cards you turn over until you observe the first red card. The random variable Y has which of the following probability distributions?

(a) the Normal distribution with mean 3

(b) the binomial distribution with $p = 0.3$

(c) the geometric distribution with probability of success 0.3

(d) the uniform distribution that takes value 1 on the interval from 0 to 3

(e) none of the above → not independent

a 12. In a large population of college students, 70% of the students own an iphone. If you take a random sample of 10 students from this population, the mean and standard deviation of the number of students in the sample who have an iphone is:

(a) μ = 7; σ = 1.4 (b) μ = 7; σ = .21 (c) μ = 7; σ = 0.49

(d) μ = ~~5.8; σ~~ = 1.4 (e) μ = ~~5.8; σ~~ = .21

binomial ✓

$\mu = np = 10(.7) = 7$

$\sigma = \sqrt{np(1-p)} = \sqrt{10(.7)(.3)}$

d 13. Which of the following is a true statement?

~~(a)~~ The binomial setting requires that there are only two possible outcomes for each trial, while the geometric setting permits more than two outcomes.

~~(b)~~ A geometric random variable takes on integer values from ~~0~~ to n.

right skew

~~(c)~~ If X is a geometric random variable and the probability of success is 0.85, then the probability distribution of X will be skewed left, since 0.85 is closer to 1 than to 0.

prob high so higher towards smaller values

(d) An important difference between binomial and geometric random variables is that there is a fixed number of trials in a binomial setting, and the number of trials varies in a geometric setting. ✓

~~(e)~~ The distribution of every binomial random variable is skewed right.

a 14. In order for the random variable X to have a geometric distribution, which of the following conditions must X satisfy?

I ~~$p < 0.5$~~

II ~~The number of trials~~ is fixed.

III Trials are independent. ✓

IV The probability of success has to be the same for each trial. ✓

V All ~~outcomes in the sample space are equally~~ likely.

(a) III and IV (b) II, III, IV, and V (c) I and III

(d) I, III, and V (e) II and III

c 15. If you buy one ticket in the Provincial Lottery, then the probability that you will win a prize is 0.11. Given the nature of lotteries, the probability of winning is independent from month to month. If you buy one ticket each month for five months, what is the probability that you will win at least one prize?

(a) 0.55

(b) 0.50

(c) 0.44

(d) 0.45

(e) 0.56

B $1 - P(0)$

$1 - \binom{5}{0}(.11)^0(.89)^5$ $P(X \geq 1)$

$1 -$ binomialpdf $(5, .11, 0)$ or cdf

$= 0.4416$

Part 2: Free Response

Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

~~16~~ 16. Let X denote the number of absences for a randomly selected student from math class in one week. Suppose that the probability distribution of X is as follows.

X	0	1	2	3	4
P(X)	0.78	0.11	0.07	0.03	0.01

(a) Find and interpret the mean (expected value) of *X*.

$0(0.78)+1(0.11)+2(0.07)+3(0.03)+4(0.01)$

$\mu_x = 0.38$ On average, a student will miss 0.38 math classes a week.

(b) Find and interpret the standard deviation of *X*.

$\sigma_x = 0.82$ On average, the amount of absences in math class will differ from the mean of 0.38 by 0.82 absences.

(c) Suppose the Mrs. Settle's pay gets docked \$1.50 for every time a student is absent from her normal pay per student is \$6. That is, P = 6 – 1.5X, where P is her pay per student. Use a linear transformation of your results in (a) and (b) to find the mean and standard deviation for P.

$\mu_p = 6-1.5(0.38) = \$5.43$ on average Mrs. Settle earns \$5.43 per student

$\sigma_p = 1.5(.82) = \$1.23$ on average Mrs. Settles pay will differ from \$5.43 by \$1.23.

~~12~~ 17. The amount of time it takes a student to do their calculus homework followed the Normal distribution with mean 25 and standard deviation 5. Choose two students independently and at random from this group.

(a) What is the expected difference in their scores? Show your work.

$P(A)-P(B)$ $\mu_A-\mu_B = 25-25 = 0$

(b) What is the standard deviation of the difference in their scores? Show your work.

$\sqrt{\sigma_A^2+\sigma_B^2} = \sqrt{5^2+5^2} = \boxed{7.07}$

(c) Find the probability that the difference in the two students' scores is greater than 6. Show your work.

$P_{A-B}(X>6) =$ normalcdf(6, 100, 0, 7.07) $\times 2$

and $P_{A-B}(X<-6)$

lower upper bounds, mean, std dev

$= 0.198 \times 2$

$= \boxed{0.396}$

18. A study shows that 66% of all dog owners greet their dog before they greet their spouse or children when they get home from work. Assume that the claim is true and each arrival home is independent.

(a) In a series of arriving home from work, what is the probability that the first time a person doesn't greet their dog first is on the 5th arrival home.

geometricpdf $(.34, 5)$ → trial #
prob doesn't greet

$(.66)^4(.34) = 0.0645$

6.45% chance that a person doesn't greet their dog first is the 5th night.

(b) What is the probability that a person greets their dog first on 4 or fewer of the arrivals home?

next 10

binomial

$P(X \leq 4)$

binomialcdf $(10, .66, 4) = 0.0836$

trials, prob greeting dog, max # of successes

8.36% chance a person greets their dog first 4 or less times out of 10.

(c) Suppose that a person goes on vacation for two weeks. When he returns, he greets his dog first only two out of ten days. Is this evidence that his rate is now less than 66%? Explain.

$P(X=2)$ binomialpdf $(10, .66, 2) = 0.0035$

trials, success rate, # of successes

0.35% chance that would happen so either the rate changed or there was extenuating circumstances from the vacation. Maybe continue to analyze before a decision is made.